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Vibration suppression of thin-walled workpiece machining considering external damping properties based on magnetorheological fluids flexible fixture

Ma Junjin, Zhang Dinghua *, Wu Baohai, Luo Ming, Chen Bing

Key Laboratory of Contemporary Design and Integrated Manufacturing Technology, Ministry of Education, Northwestern Polytechnical University, Xi'an 710072, China

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Abstract Milling of the thin-walled workpiece in the aerospace industry is a critical process due to the high flexibility of the workpiece. In this paper, a flexible fixture based on the magnetorheological (MR) fluids is designed to investigate the regenerative chatter suppression during the machining. Based on the analysis of typical structural components in the aerospace industry, a general complex thin-walled workpiece with fixture and damping constraint can be equivalent as a rectangular cantilever beam. On the basis of the equivalent models, natural frequency and mode shape function of the thin-walled workpiece is obtained according to the Euler–Bernoulli beam assumptions. Then, the displacement response function of the bending vibration of the beam is represented by the product of all the mode shape function and the generalized coordinate. Furthermore, a dynamic equation of the workpiece-fixture system considering the external damping factor is proposed using the Lagrangian method in terms of all the mode shape function and the generalized coordinate, and the response of system under the dynamic cutting force is calculated to evaluate the stability of the milling process under damping control. Finally, the feasibility and effectiveness of the proposed approach are validated by the impact hammer experiments and several machining tests.

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1. Introduction

In aerospace industry, the machining vibration of thin-walled workpiece is an undesirable phenomenon. The analysis of the negative influence caused by machining vibration, especially for machining flexible workpiece such as aeroengine blades, casings, impellers, blisks, has received special attention due to its effects on the accuracy of the final workpiece, the production efficiency and the life of machine spindles and cutters. Milling process, which plays a critical role in manufactur-

* Corresponding author.
E-mail addresses: majunjin@gmail.com (J. Ma), dhzhang@nwpu.edu.cn (D. Zhang), wubaohai@nwpu.edu.cn (B. Wu).

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ing, has been extensively used to the finishing machining of the low rigidity and large machining deformation workpiece, and the machining vibration is a serious problem in machining. Therefore, it is necessary to investigate the machining vibration dynamic response of the thin-wall workpiece to suppress vibration in machining.

For the efforts of investigating the milling vibration of thin-walled flexible workpiece, reasonable cutting parameters should be selected in machining according to chatter stability diagram. Recently, in the aspect of chatter stability prediction, numerous theoretical and experimental investigations have been made in model prediction, analysis, and various designs of fixture devices and so on. To accurately study the machining vibration, the prediction and analysis of the dynamic machining stability of the thin-walled workpiece is a fundamental problem. Many research works about this aspect have been developed. Altintas and Budak[1,2] presented an analytical method (ZOA method) based on the dynamic milling coefficients approximated by their Fourier series to predict the milling stability lobes. Merdol and Altintas[3] overcome the existence of additional stability regions and period doubling bifurcations in small radial cut depth during milling process. Zhou et al.[4] proposed an analytical model considering tool-workpiece engagement region and different cutter lead angles for chatter stability prediction in bull-nose end milling of aeroengine casings, and the predicted method is well agreed with experimental results. Insperger and Stépán et al.[5–7] proposed the semi-discretization method to efficiently conduct stability analysis to the linear milling delayed systems. Considering the calculated efficiency, Ding et al.[8,9] presented the full-discretization methods to predict milling stability based on different mathematic theory. Then, this method is extended by Liu et al.[10] based on the Hermite interpolation and the Floquet theory.

In machining vibration suppression, many different passive and active control methods are investigated. Zhang et al.[11] presented a piezoelectric active vibration control method that increases the damping of the thin-walled flexible workpiece to mitigate the milling vibration. Rashid et al.[12] adopted an active controlled palletised workholding system, which is different from traditional approach in machining systems to dampen the unwanted vibration. However, many considerable piezoelectric devices and external system in their method, which take much time and is not suitable for complex parts. Sathianarayanan et al.[13] and Mei et al.[14,15] proposed the method of using magnetorheological fluids to adjust the stiffness of boring bar, which can efficiently improve the stability of boring bar and reduce machining vibration. However, they only investigated the effects of the damping material on the boring bar and the properties of the whole fixture are not considered. Shamoto et al.[16] presented a new method of simultaneous double-sided milling to machine flexible plates and the chatter vibration is efficiently suppressed by the experimental validation. Kolluru et al.[17] presented a novel surface damping solution which took distributed discrete masses as viscoelastic layer to large thin-walled casings for reducing the machining vibration obviously. Zhang et al.[18] proposed a component synthesis active vibration suppression method based on zero-place-ment technique to suppress the vibration of flexible systems. Yang et al.[19] designed a vibration suppression device for the thin-walled flexible workpiece based on the electromagnetic induction principle, and the excitation tests and machining tests are carried out to verify the efficiency of the device. Zeng et al.[20] proposed a novel fixture design approach to suppress the machining vibration of flexible workpiece. In this work, appropriate fixture layout scheme concerned with suppressing the machining vibration of the flexible workpiece is designed based on the proposed dynamic model of workpiece-fixture-cutter system. Later, Wan et al.[21,22] presented a new fixture layout optimization method for the milling of thin-walled multi-framed workpiece to obtain better fixture supports scheme, which took the machining quality and suppression of machining vibration into consideration, in addition, the predicted and experimental results show that the reasonable fixture design can efficiently improve the machining accuracy and significantly suppress machining vibration of flexible workpiece. However, those works only focus on the properties of the fixture support and the effect of the damping properties of the whole fixture on the dynamic machinability is ignored.

Notwithstanding many valuable results aforementioned, current researches on chatter stability prediction and vibration control are mainly concentrated on the effect of the cutting process parameters, while the actual dynamic response properties of the machining system depended on the whole damping properties of the system according to the dynamic differential equation to some extent. However, the dynamic damping properties are not considered in these studies. To address this issue, a semi-active control flexible fixture based on MR fluids is proposed for the complex thin-walled workpiece machining in this paper, which takes the aforementioned missing damping factor into consideration.

Henceforce the paper is organized as follows: Section 2 describes the problem formulation of the thin-walled workpiece in machining considering the equivalent model of the thin-walled workpiece. Then, natural frequency and mode shape function of the thin-walled workpiece based on the Euler–Bernoulli beam assumptions is calculated in Section3 3. Section 4 constructs the dynamic equation of the workpiece-fixture system considering the external damping factor, and the response of system under the dynamic cutting force is calculated to evaluate the stability of the workpiece-fixture system. Milling experimental tests are carried out for validating the feasibility and efficiency of the proposed chatter theory and the flexible fixture, and the experimental results is discussed in Section 5. Following the discussion, some conclusions and future works are summarized in the end.

2. Problem formulation

The machining of thin-walled workpiece is critical to obtain high precision and good quality of the workpiece in aerospace industry. Due to the typical structure of the thin-walled workpiece used in the aerospace industry, the parts with fixture constraint can be approximately equivalent as a rectangular cantilever beam as shown in Fig. 1. Firstly, a solid workpiece with lower precision and larger scattered errors is fixed on the working table of machine tools by the fixture, which can keep the correct position between the cutting tools and the workpiece, and an amount of materials is removed for the required final structure in machining process. In this process, the two stages are divided. One is rough machining, the workpiece with big cutting allowance and the cutter with shank of tool can be regarded as rigid, the cutter with short cutting tool can be

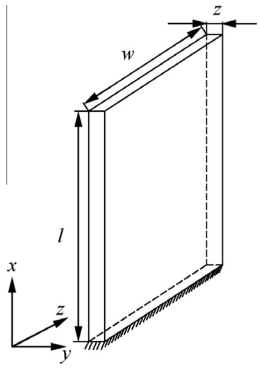

Fig. 1 A rectangular plate representing thin-walled workpiece.

regarded as rigid. Therefore, the large spindle speed and depth of cut are used to improve the material removal efficiency. The other stage is finishing machining. At this stage, the desired geometrical and surface integrity of the thin-walled workpiece can be obtained. During the finishing process, the short cutting tool can be regarded as rigid and the workpiece has the thin-walled properties and is considered as flexible, therefore, the machining vibration is a dominant factor influencing the dynamic machinability.

In machining, due to the bad rigidity and weak intensity of the thin-walled plate, when the cutting tool passes through the surface to be machined, the large machining vibration can influence the machining efficiency and deteriorate the surface accuracy. To address this problem, a semi-active control flexible fixture based on MR fluids is used for the thin-walled plate machining considering damping factor, and the workpiece-fixture-cutter system is shown in Fig. 2. To investigate an approximate dynamic response analysis solution, the complex thin-walled workpiece is simplified as a uniform rectangular thin-walled plate (the length is $0 < x < l$, the width $w \ll l$, and the thickness of the plate $z \ll w$), which is shown in Fig. 1.

In Fig. 2, a flexible fixture based on the magnetorheological (MR) fluids could be designed by adding the additional damping property to the machining system. Under the tunable magnetic field intensity H, the suspended magnetic particles in the container align into chains along the direction of external magnetic field. Then, the MR fluids transmit from a liquid state to a nearly solid state in millisecond and change back to the liquid state when the external magnetic field is removed, which is defined as magnetorheological effect. During this process, the

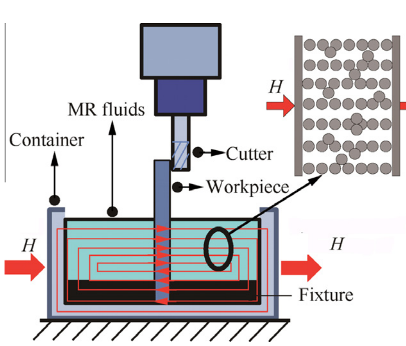

Fig. 2 Workpiece-fixture-cutter system based on magnetorheological fluids.

shear stress of MR fluids sharply increases, which can provide adequate damping support force for the thin-walled workpiece contacted with the MR fluids in machining. Therefore, the rigidity of the thin-walled workpiece is largely improved. In the machining process, due to the flexibility of the thin-walled workpiece, the workpiece is prone to produce vibrations and deformation as shown in Fig. 3, the dynamic displacement of the workpiece is equal to the deformation of the magnetic particle chains, and the thin-walled workpiece in the MR fluids is subjected to the action of the evenly distributed damping force, which improve the stiffness of the thin-walled workpiece and effectively suppress machining vibration.

3. Natural frequency and mode shape functions of thin-walled workpiece

The thin-walled workpiece is modeled as a continuous, orthotropic rectangular thin-walled plate with the length l, width w, uniform thickness z (Fig.1). Due to the width of the rectangular plate workpiece $w \ll l$, the uniform rectangular thin-walled plate can be regarded as an Euler–Bernoulli beam to approximately obtain its natural frequency and mode shape functions. Assumed the length of the beam is l, ρ is the mass unit length and EI is the flexural rigidity of the cantilever beam. Therefore, the coordinate system is shown in Fig. 4, then, the infinitesimal section dx is selected for analysis, x_1 is the distance from bottom to the infinitesimal section dx, and Q, M represent the moment and torque on the infinitesimal section dx. Based on the Euler–Bernoulli beam assumptions, for the infinitesimal section dx in force analysis, the motion equation of infinitesimal section dx in free vibration along y direction can be expressed as

$$\rho \mathrm{d}x \frac{\partial^2 y}{\partial t^2} = \frac{\partial Q}{\partial x} \tag{1}$$

In moment analysis, the influence of the moment of inertia is neglected, so the moment equilibrium method is used to analyze the infinitesimal section dx.

$$Q = EI \frac{\partial}{\partial x}\left(\frac{\partial^2 y}{\partial x^2}\right) \tag{2}$$

By inserting Eq. (2) into Eq. (1), the motion equation of the bending vibration of the beam in free vibration can be obtained as

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right) + \rho \frac{\partial^2 y}{\partial t^2} = 0 \tag{3}$$

Considering the boundary conditions of cantilever beam, the natural frequency ω_i of the cantilever beam can be derived from Eq. (3)

$$\omega_i = \sqrt{\alpha_i^4 EI/\rho} \tag{4}$$

where

$$\alpha_i = (2i-1)\pi/(2l) \qquad i = 1, 2, 3, \ldots, n$$

And all the mode shape functions corresponding to its natural frequency ω_i are represented as

$$X_i(x) = \cosh \alpha_i x - \cos \alpha_i x + \beta_i(\sinh \alpha_i x - \sin \alpha_i x) \tag{5}$$

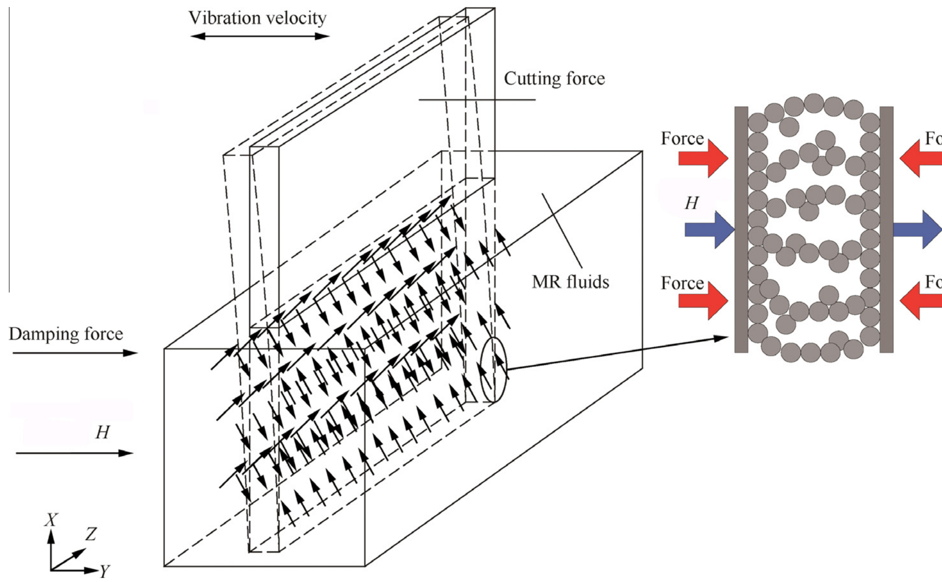

Fig. 3 Damping properties of MR fluids generated by current and machining vibration.

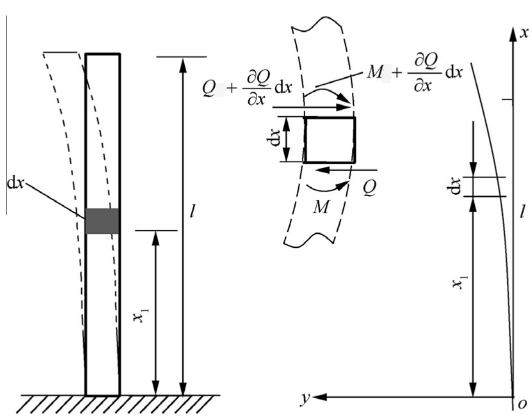

Fig. 4 Bending vibration of beam.

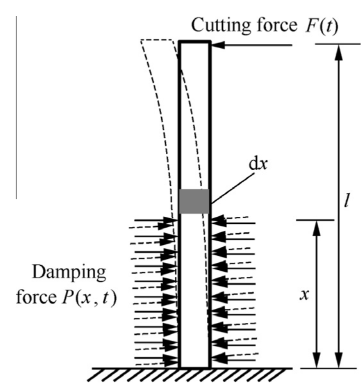

Fig. 5 Bending vibration of beam under dynamic cutting and damping force.

where

$$\beta_i = -\frac{\sinh \alpha_i l - \sin \alpha_i l}{\cosh \alpha_i l + \sin \alpha_i l}$$

4. Dynamic response of thin-walled workpiece with MR fluids' damping constraint

When the dynamic cutting force $F(t)$ and the damping force $P(x,t)$ generated by MR fluids are applied on the cantilever beam as shown in Fig. 5, the motion equation of the bending vibration of the beam can be written as

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}(x,t)\right) + \rho(x)\frac{\partial^2 y}{\partial t^2}(x,t) = F(t) + P(x,t) \tag{6}$$

For simplification of expression in the later Section, the solution of Eq. (6) can be expressed as

$$y(x,t) = \sum_{i=1}^{n} X_i(x)q_i(t) = \boldsymbol{X}_i\boldsymbol{q}(t) \ \ i = 1,2,3,\ldots,n \tag{7}$$

where $X_i(x)$ contained in $\boldsymbol{X}_i = [X_1, X_2, X_3, \cdots, X_i, \cdots, X_n]$ represents mode shape function of the bending vibration of the beam, and $q_i(t)$ contained in $\boldsymbol{q} = [q_1, q_2, \cdots, q_i, \cdots, q_n]^{\mathrm{T}}$ is the generalized coordinate of the system.Using Eq. (7), the kinetic energy E_{k} of the dynamic cutting force vibration of the thin-walled workpiece can be obtained as

$$E_{\mathrm{k}} = \frac{1}{2}\int_0^l \rho(x)\left[\frac{\partial y}{\partial t}(x,t)\right]^2 \mathrm{d}x = \frac{1}{2}\dot{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{M}\dot{\boldsymbol{q}} \tag{8}$$

where each element of the generalized velocity $\dot{q}_i$ is $\dot{q}_i = \partial y/\partial t$, and each element of the generalized mass matrix $\boldsymbol{M}$ corresponding to the generalized coordinate is given by

$$M_{ij} = \int_0^l \rho(x)[X_i(x)]^2 \mathrm{d}x \ \ i,j = 1,2,\ldots,n \tag{9}$$

For the cantilever beam, the bending potential energy E_{p} is only considered to simplify the complexity of fixture-workpiece system, which can be represented by

$$E_{\mathrm{p}} = \frac{1}{2}\int_0^l EI(x)\left[\frac{\partial^2 y}{\partial x^2}(x,t)\right]^2 \mathrm{d}x = \frac{1}{2}\boldsymbol{q}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{q} \tag{10}$$

where each element of the generalized stiffness matrix $\boldsymbol{K}$ corresponding to the generalized coordinate is defined as

$$K_{ij} = \int_0^l EI(x)[\ddot{X}_i(x)]^2 \mathrm{d}x \; i,j = 1,2,\ldots,n \tag{11}$$

Due to the damping properties of the MR fluids and the beam, the flexible fixture-workpiece system can be regarded as a dissipative system. The MR fluid is damping material, which can change its rheological behavior under the different current to improve the stiffness of the thin-walled workpiece. Thus, the uniform distributed damping force per unit length to the cantilever beam can be expressed as

$$\begin{cases} p_1(x,t) = c_1 \frac{\partial y}{\partial t} = c_1 \sum_{i=1}^{n} X_i(x)\dot{q}_i(t) \\ p_2(x,t) = c_2 \frac{\partial y}{\partial t} = c_2 \sum_{i=1}^{n} X_i(x)\dot{q}_i(t) \end{cases} \tag{12}$$

where c_1 and c_2 are the damping constant of the MR fluids and structural damping constant of the cantilever. Therefore, when the vibration occurs, the energy loss E_l is approximated by the work done of the damping force corresponding to the generalized vibration velocities.

$$\begin{aligned} E_l &= P(x,t)\dot{q}_i(t) = \left[\int_0^a p_1(x,t)\mathrm{d}x + \int_0^l p_2(x,t)\mathrm{d}x\right]\dot{q}_i(t) \\ &= \sum_{i=1}^{n}\left[\int_0^a c_1 X_i^2(x)\mathrm{d}x + \int_0^l c_2 X_i^2(x)\mathrm{d}x\right]\dot{q}_i^2(t) \\ &= \dot{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{C}\dot{\boldsymbol{q}} \qquad 0 < a < l \end{aligned} \tag{13}$$

where each element of the generalized damping matrix $\boldsymbol{C}$ corresponding to the generalized coordinate is expressed as

$$C_{ij} = \int_0^a c_1 X_i^2(x)\mathrm{d}x + \int_0^l c_2 X_i^2(x)\mathrm{d}x \; i,j = 1,2,\ldots,n \tag{14}$$

In machining process, using the classical virtual work principle, the work done due to dynamic cutting force is

$$\delta W = \int_0^l F(x,t)\left[\sum_{i=1}^{n} X_i(x)\delta q_i(t)\right]\mathrm{d}x = \sum_{i=1}^{n} Q_i \delta q_i(t) \tag{15}$$

In Eq. (15), Q_i is defined as the generalized force, which is given by

$$Q_i = \int_0^l F(t)X_i(x)\mathrm{d}x \tag{16}$$

In order to obtain the motion equation of the generalized coordinate q_i, the Lagrangian equation of the thin-walled workpiece with MR fluids damping material constraint can be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E_k}{\partial \dot{\boldsymbol{q}}}\right) - \frac{\partial E_k}{\partial \boldsymbol{q}} + \frac{\partial E_l}{\partial \dot{\boldsymbol{q}}} + \frac{\partial E_p}{\partial \boldsymbol{q}} = Q_i \tag{17}$$

where $Q_i = \sum_{i=1}^{n} \int_0^l F(t)X_i \mathrm{d}x$.

By substituting Eq. (8), (10), (13) and (16) into Eq. (17), the equations of motion of the thin-walled workpiece with MR fluids damping material constraint can be obtained as

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{Q}_i \tag{18}$$

Considering the complexity of vibration damping system, for simplification, the damping matrix is regarded as a linear combination of mass and stiffness matrix, so a proportional damping is used to avoid cumbersome mathematical calculation:

$$\boldsymbol{C} = \alpha\boldsymbol{M} + \beta\boldsymbol{K} \tag{19}$$

where α and β are constants.

Therefore, the Eq. (18) can be rewritten as

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + (\alpha\boldsymbol{M} + \beta\boldsymbol{K})\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{Q}_i \tag{20}$$

In order to decouple the motion differential equations (20), the solution vector $\boldsymbol{q}$ can be expressed as a linear combination of the natural modes of the undamped system, as in the case of Eq. (7), and $\boldsymbol{q}(x,t) = \sum_{i=1}^{n} X_i(x)Y_i(t) = \boldsymbol{X}_i(x)\boldsymbol{Y}(t)$, $i = 1,2,3,\ldots,n$, where $\boldsymbol{Y} = [Y_1, Y_2, \cdots, Y_i, \cdots, Y_n]^{\mathrm{T}}$ is the generalized coordinates. Thus, Eq. (20) can be rewritten as

$$\boldsymbol{M}\boldsymbol{X}_i\ddot{\boldsymbol{Y}}(t) + (\alpha\boldsymbol{M} + \beta\boldsymbol{K})\boldsymbol{X}_i\dot{\boldsymbol{Y}}(t) + \boldsymbol{K}\boldsymbol{X}_i\boldsymbol{Y}(t) = \boldsymbol{Q}_i \tag{21}$$

In addition, the element of matrix $\boldsymbol{X}_i$ is normalized[23] and $\boldsymbol{X}_i^{\mathrm{T}}$ is pre-multiplied by Eq. (21). Then, Eq. (21) can be decoupled as

$$\ddot{\boldsymbol{Y}}(t) + 2\zeta_i\omega_i\dot{\boldsymbol{Y}}(t) + \omega_i^2\boldsymbol{Y}(t) = \boldsymbol{X}_i^{\mathrm{T}}\boldsymbol{Q}_i \tag{22}$$

where ω_i is the ith order natural frequency of the undamped system, and $2\zeta_i\omega_i = \alpha + \beta\omega_i^2$, ζ_i represents mode damping ratio corresponding to the ith order natural mode shape under the external MR fluids damping materials. According to Eq. (22), each response of the cantilever beam vibration system with the MR fluids damping constraint can be solved as a viscously damped single degree of freedom system. Therefore, the response of the bending vibration of the beam under dynamic cutting and damping force is expressed as

$$\begin{aligned} q(x,t) &= \sum_{i=1}^{n} X_i(x)Y_i(t) \\ &= \sum_{i=1}^{n} X_i(x)\left\{\mathrm{e}^{-\zeta_i\omega_i t}\left[\cos\omega_{\mathrm{d}i}t + \frac{\zeta_i}{\sqrt{1-\zeta_i}}\sin\omega_{\mathrm{d}i}t\right]Y_0(0)\right. \\ &\quad + \left[\frac{1}{\omega_{\mathrm{d}i}}\mathrm{e}^{-\zeta_i\omega_i t}\sin\omega_{\mathrm{d}i}t\right]\dot{Y}_0(0) \\ &\quad \left. + \frac{1}{\omega_{\mathrm{d}i}}\int_0^t X_i Q_i(\tau)\mathrm{e}^{-\zeta_i\omega_i(t-\tau)}\sin\omega_{\mathrm{d}i}(t-\tau)\mathrm{d}\tau\right\} \end{aligned} \tag{23}$$

where $\omega_{\mathrm{d}i} = \omega_i\sqrt{1-\zeta_i^2}$, $i = 1,2,3,\ldots,n$.

For the cantilever beam, the initial displacement $Y_0(0)$ and velocity $\dot{Y}_0(0)$ are zero. Therefore, the 1st mode dynamic response of the workpiece-fixture system under dynamic cutting force on the top of the equivalent rectangular cantilever beam is calculated as

$$q(x,t) = \frac{1}{\omega_{\mathrm{d}1}}\int_0^t X_1(x)Q_1(\tau)\mathrm{e}^{-\zeta_1\omega_1(t-\tau)}\sin\omega_{\mathrm{d}1}(t-\tau)\mathrm{d}\tau \tag{24}$$

5. Experimental results and discussion

According to the above analysis, a flexible fixture using MR fluids is designed to exert extra damping action on the thin-walled workpiece. Without loss of generality, the thin-walled plate with the elastic edges and the MR damping support is used to represent approximately the complex thin-walled workpiece. The schematic illustration of the flexible fixture and the thin-walled workpiece studied in this research is shown in Fig. 2. The purpose of the study is to design an appropriate flexible fixture considering the vibration characteristics of the combined workpiece-fixture-cutter system. To validate the effectiveness of the designed flexible fixture, both the impact experiments and the machining experiments are implemented on the three-axis milling center (YHVT8507) and the experimental tests are carried out (see Fig. 6).

5.1. Impact tests

The impact hammer experiment is a general way of experimental analysis for the vibration characteristics of the combined workpiece-fixture system. The test system are model hammer (Kistler, 500 N), acceleration sensors (Dytran, 3325F1-16847, Ref. sensitivity 10.25 mV/g.), SIRIUS ACC data acquisition instrument (DEWESoft-SIRIUS *i* series), and computer. The impact hamper is used as an excitation source for the measurement of frequency response function, and an accelerometer is selected to measure the response of the workpiece-fixture system. Then, the measured vibration response modes can be used to compare and analyze the vibration characteristics of different dampers acted on the workpiece. The dimension of the flexible thin-walled workpiece made from 45 steel is 130 mm × 60 mm with the thickness of 5 mm. The Young's modulus of the material is $E = 2.10 \times 10^{11}$ Pa. The Poisson's ratio υ is 0.269, and the density of the material is 7850 kg/m^3. The bottom of the thin-walled workpiece is fixed at the bottom of the MR fluids container, and the distributed points *A*, *B*, *C*, *D*, *E*, *F* on the thin-walled workpiece as shown in Fig. 7 are chosen for impact tests, where point *A* and *B* are symmetric with respect to *C*, and *D* and *F* are symmetric with respect to *D*.

As you know, different impact points can obtain different response of the thin-walled workpiece. Therefore, for verifying the effectiveness and feasibility of the MR fluids flexible fixture, the impact responses on the distributed points *A*, *B*, *C*, *D*, *E*, *F* on the thin-walled workpiece are all obtained to analyze the vibration properties. The response at point *A* is approximately equal to that at point *C* due to the symmetric distribution, and the response at point *D* is similar to the

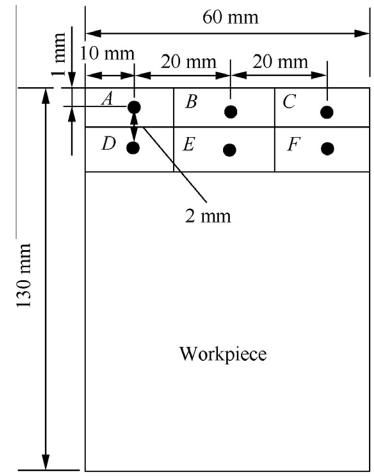

Fig. 7 Distributed points *A*, *B*, *C*, *D*, *E*, *F* on thin-walled workpiece for impact tests.

response at point *F* for the same reason. However, the points *B* and *E* are located in the middle of the thin-walled plate and the response are different from points *A*, *C* and *D*, *F* on the collinear, respectively. In order to analyze the impact response problems of MR fluids flexible fixture easily, point *A* is selected as the typical position for the vibration response because it is located near the cutter entry point, which is sensitive to dynamic cutting force. Thus the modal parameters of workpiece on the point *A* are listed in Table 1.

In order to exactly analyze the difference dynamic properties of the workpiece-fixture system with and without damping control, the frequency response functions at different damping parameters are plotted in Fig. 8. It is observed that both the amplitudes of the frequency responses functions and the values of the frequency of the workpiece-fixture system are similar with and without MR fluids damping material under 1 A current in first vibration mode. Moreover, the amplitude values of the system with damper under 2 A and 3 A current reduce obviously, and the frequency response under MR fluids damping material under 2 A and 3 A current are about 390.6 Hz. In addition, all the higher vibration modes are

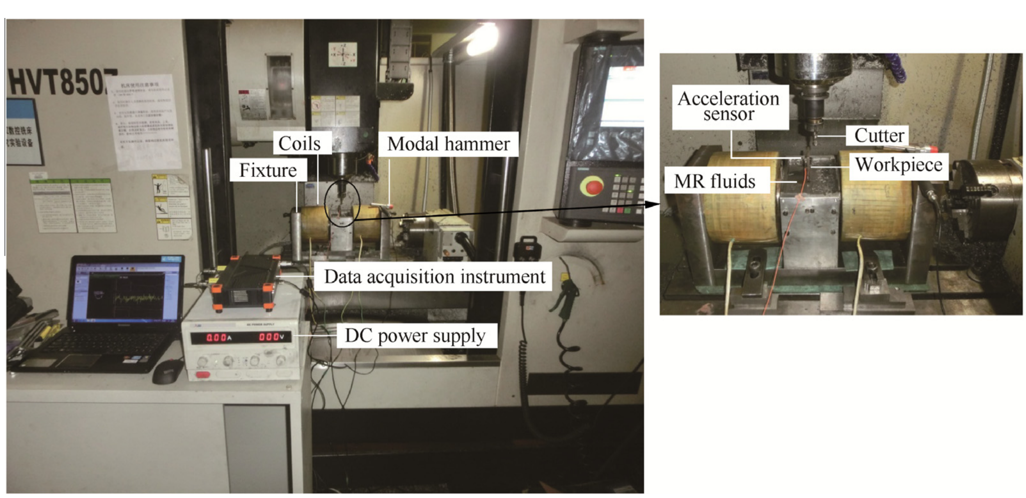

Fig. 6 Machining tests.

Table 1 Modal parameters of thin-walled workpiece at point *A*.

Item	Current (A)	Natural frequency (Hz)	Damping ratio ζ	Stiffness (N/m)
Without damper	0	361.328	0.016	3.9970×10^7
With damper	1	380.859	0.025	4.4408×10^7
	2	390.625	0.036	4.6715×10^7
	3	395.508	0.047	4.7890×10^7

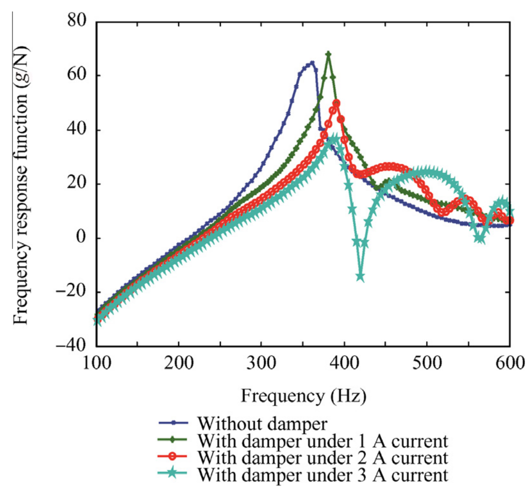

Fig. 8 Frequency response functions of workpiece-fixture system with and without damper.

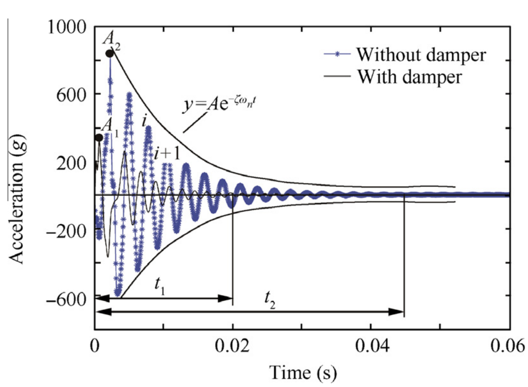

Fig. 9 Comparison of the vibration properties tests with and without damper.

obviously suppressed under the action of the MR fluids damping material. The whole experimental results validate that the MR fluids flexible fixture can improve the rigidity of thin-walled workpiece and suppress the machining vibration than the traditional fixture without MR fluids damping material.

Furthermore, Fig. 9 shows the experimental comparisons of the impact vibration characteristics of the workpiece-fixture system with and without the damping control, and Table 2 lists the peaks of the impact modal parameters for the thin-walled workpiece measured using the impact tests with and without damper.

Table 2 Modal parameters of workpiece system.

Item	Damping ratio ζ	Peak values (*g*)				
		1	2	3	4	5
Without damper	0.016	832.4	599.8	400.1	266.2	177.6
With damper	0.047	362.8	238.4	140.8	82.19	51.59

In Fig. 9, it is noted that the amplitude values of peaks with damper ($A_1 = 362.8\,g$) are smaller than that without damper ($A_2 = 832.4$ g), and the system decrement with and without damper is approximately consistent with exponential function $y = Ae^{-\zeta\omega_n t}$ as the time changes. Furthermore, the decrement time from external excitation to stable state with the MR fluids damping materials is $t_1 = 0.02$ s, which is smaller than that $t_2 = 0.045$ s without the action of the MR fluids damping materials. In addition, under the hysteresis damping, the motion of the excitation can be considered to be nearly harmonic (the energy loss per cycle is small), and the decrease in amplitude per cycle can be determined using energy balance principle. Therefore, the amplitude ratio of two adjacent wave peaks i, $i+1$ is a constant and can be derived as $A_i/A_{i+1} = (2+\pi\alpha)/(2-\pi\alpha)$ ($\alpha = p/k$ is a dimensionless damping constant, p is hysteresis damping constant.), and the amplitude ratios without and with damper are about 1.5 and 1.6 according to Table 2, respectively.

5.2. Machining tests

In order to further verify the rationality and feasibility of the MR fluids flexible fixture, machining experiments are carried out on the machining center with and without the MR damping control. Before machining, milling stability is used to predict the effectiveness of the MR fluids' damping fixture. Because the natural frequency of the workpiece-fixture system under the action of MR fluids damping is not equal to the tooth passing frequency, the milling stability lobe diagram is used to select the different cutting parameters to validate the milling stability. Then, the stability lobe diagram for the thin-walled workpiece in three-axis milling with different damping ratios can be calculated using the proposed model as shown in Fig. 10.

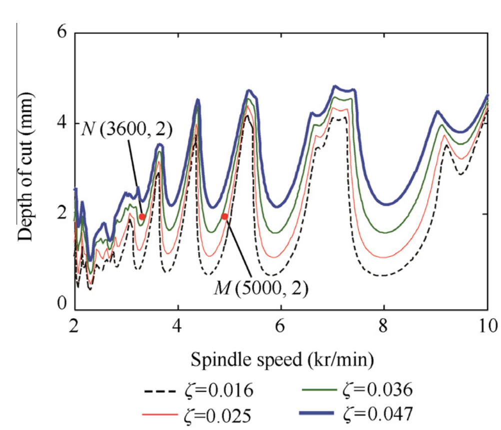

Fig. 10 Stability lobe diagrams at different damping ratio (Point *M* and *N* are selected for machining tests).

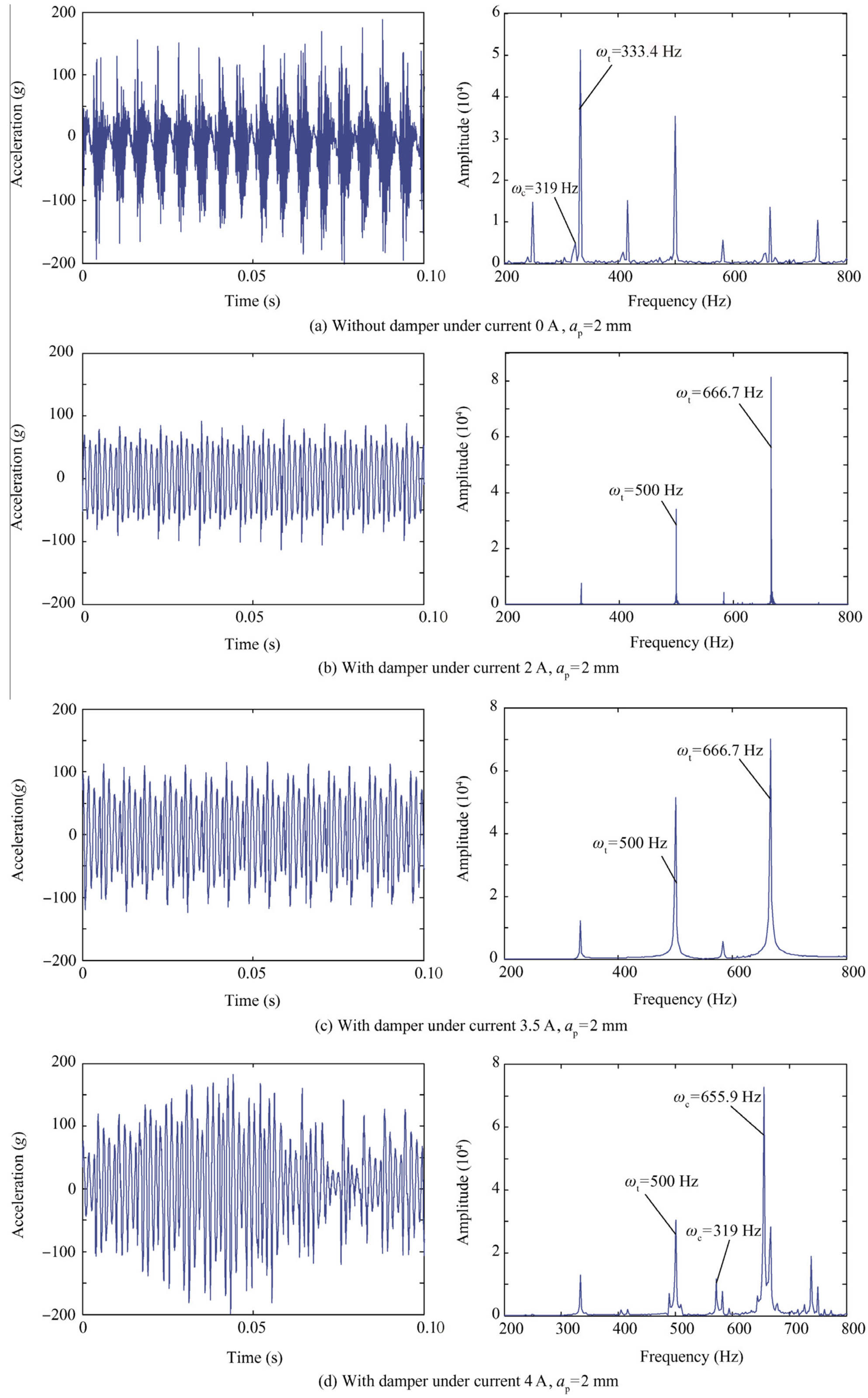

Fig. 11 Machining vibration acceleration response of thin-walled plate (S = 5000 r/min, F = 500 mm/min, a_e = 0.3 mm).

In Fig. 10, it can be observed that stability lobe charts show remarkable variation with respect to the spindle speeds and the depth of cut with and without damping control, and the milling stability region is obviously enlarged under the MR fluids damping control in vertical direction at different damping ratios and does not change in the horizontal direction with the damping ratio increasing, which is used to validate the feasibility of the MR fluids flexible fixture in milling vibration suppression. In the fixed milling system, the larger the damping ratio under the action of the MR fluids on the thin-walled workpiece is, the more obviously stable machining is.

In machining, the dimension of the flexible workpiece is 130 mm × 60 mm with the thickness of 5 mm. The cylindrical helical milling cutter with diameter 10 mm and the number of teeth $N = 2$ is selected according to the principle of minimizing the effects of the cutter vibration on tool path. Then, spindle speed $S = 5000$ r/min, radial cutting depth $a_e = 0.3$ mm and depth of cut in axial direction $a_p = 2$ mm (point M, and the stability lobes is shown in Fig. 10.) are selected for validating the damping properties of the MR fluids under different currents. When the MR damping materials does not acted on the thin-walled workpiece, the maximum instantaneous acceleration is about 150 g as shown in Fig. 11 (a), and the fast Fourier transformation (FFT) method is used to analyze the time domain singles. In Fig. 11(a), it is found that different harmonic frequency ω_c such as 403.4 Hz, 820.1 Hz and 1156 Hz generated during the milling besides the tooth passing frequency ω_t 166.7 Hz together with its integer multiples (i.e., 333.4, 516.5, 500, 666.7, 833.4, 1000 Hz). Therefore, the milling process is unstable and the waviness caused by chatter is printed on the machined surface. Later, when the MR fluids damping materials contacting the workpiece is under the magnetic field, the maximum acceleration reduces to about 60 g with the damping material action under $I = 2$ A as shown in Fig. 11(b). It is noted that the vibration acceleration of thin-walled plate reduces by nearly 30% compared with that without damping control. The reason for this is that the damping force generated by the magnetorheological effect of MR fluids at different electric currents can significantly improve the dynamic machinability of the thin-walled workpiece, and the milling process is stable with the electric current increasing. In addition, the frequency spectrum also denotes the stable machining process as the tooth passing frequency and its harmonics dominate the whole machining under $I = 2$ A. In Fig. 11(c), the milling process still keeps stable under $I = 3.5$ A. However, the value of acceleration amplitude raised to about 120 g and the quality of the machined surface deteriorate as the electric current increases, which reaches the chatter frequency ω_c (i.e., 571.5, 655.9, 738.3, 822.7 Hz.) as shown in Fig. 11(d). The main reason for this is that the magnetic particles of the MR fluids have the limit of saturation magnetization, and when the magnetic field strength exceeds the critical value of saturation magnetization, the shear thinning phenomenon of MR fluids works gradually and the damping effect of MR fluids reduces sharply.

To give a clear comparison between the machining vibration of the thin-walled plate with and without damping control, cutting parameters ($S = 3600$ r/min, $F = 500$ mm/min, radial cutting depth $a_e = 0.3$ mm, axial cutting depth $a_p = 2$ mm) are chosen according to the stability lobes for machining tests. Fig. 12 shows the predicted and measured vibration displacement response of the thin-walled plate at the selected points A for the two cases with and without damping control. It can be noted that the maximum values of vibration displacement of point A on the thin-walled workpiece with the length 129 mm and the width 10 mm are 0.453, 0.312 0.143 mm under the different currents 0, 1, 2 A. Moreover, an agreement of the measured and predicted displacement response of the point A is reached in milling process with

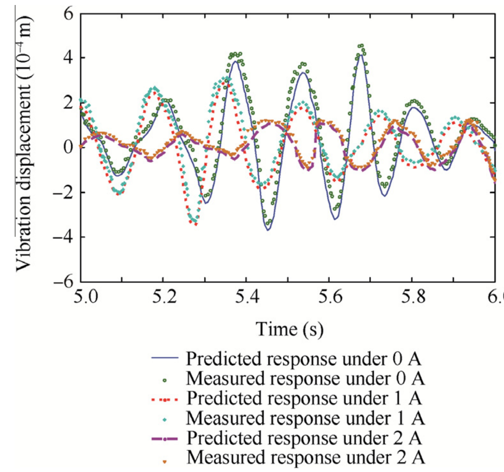

Fig. 12 Comparison of predicted and measured displacement responses of selected response point A with and without damper (Cutting parameters: $S = 3600$ r/min, $F = 500$ mm/min, $a_e = 0.3$ mm, $a_p = 2$ mm).

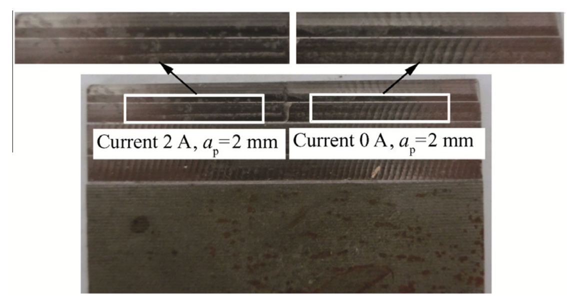

Fig. 13 Machining vibration of whole thin-walled sheet milling process($S = 3600$ r/min, $F = 500$ mm/min, $a_e = 0.3$ mm).

and without damper control. However, the difference existing among the displacement values with and without damper control indicates that the damping property is a crucial factor influencing the dynamic behavior of the thin-walled workpiece in machining. And, the experimental results validate that the proposed equivalent dynamic models can effectively predict the dynamic response of the thin-walled workpiece during machining. In addition, the surface waviness of vibration in machining can fully reflect on the machined surface, so the machining vibration can be detected by observing the quality of the machined surface as shown in Fig. 13. It is found that the quality of the machined surface with damping control is better than that without damper control obviously, which further indicates that the method of using the MR fluids flexible fixture to suppress the vibration of the thin-walled workpiece in machining is feasible and effective.

6. Conclusions

(1) An equivalent model of the typical complex thin-walled structure in the aerospace industry is proposed based on the Euler–Bernoulli beam assumptions for simplification, which is convenient to consider the MR fluids damping properties.

(2) A dynamic equation of the workpiece-fixture system considering the external MR fluids damping constraint is proposed using the Lagrangian method, and the response of system under the dynamic cutting force is calculated to evaluate the stability of the milling process under damping control compared with the response without damping control.
(3) The proposed model and approach are validated by the experimental results, which are obtained by the modal and machining tests with and without the action of MR fluids damping materials. And a good agreement has been reached in machining response control.
(4) The proposed equivalent model only treats the thin-walled plate as cantilever beam structure, and the bending potential energy is considered in the model, neglecting the torsional potential energy. In machining, the MR fluids' damping material plays an important role in improving the dynamic properties of the thin-walled workpiece, which is only validated in three-axis milling of the complex thin-walled workpiece based on Euler–Bernoulli beam assumptions. Therefore, for the complex thin-walled structure workpiece (i.e., blades, casings, impellers, blisks.) or five-axis milling, the comprehensive equivalent dynamic model and the new flexible fixture are necessary in the future because the dynamic responses in all directions determine the regenerative chatter in machining.

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Ma Junjin received his B.S. and M.S. degrees from Henan Polytechnic University in 2009 and 2012, respectively, and is now a Ph.D. candidate from Northwestern Polytechnical University. His main research interests are multi-axis machining and high-speed machining.

Zhang Dinghua received his B.S., M.S. and Ph.D. degrees from Northwestern Polytechnical University in 1981, 1984 and 1989, respectively, and became a professor there in 1991. His main research interests are multi-axis machining, high-speed machining, machining surface integrity and industrial CT technology.

Wu Baohai received the B.S., M.S., and Ph.D. degrees from Xi'an Jiaotong University in 1997, 2000, and 2005, respectively. He is currently an Associate Professor in Northwestern Polytechnical University, and his research interests include multi-axis NC machining, CAD/CAM, and smart machining technology.